Bandwidth and Propagation Delay Exercises ANS

Lecture 3, Spring 2026

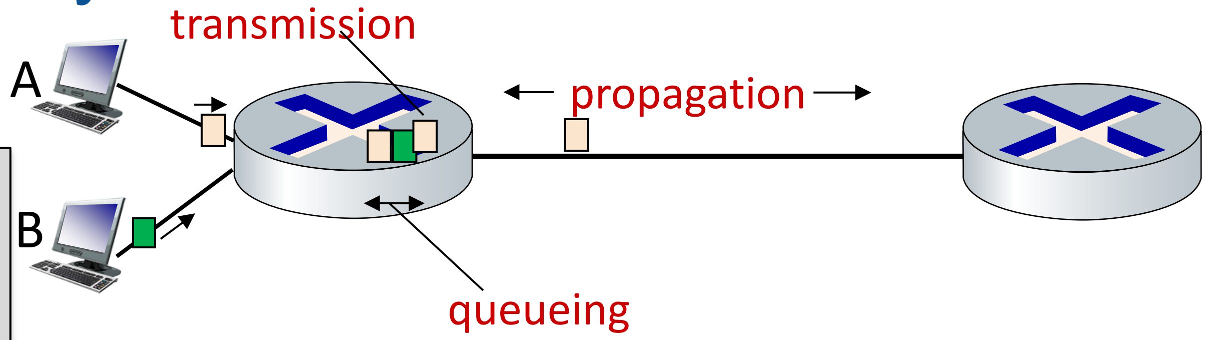
Links

- Bandwidth and Propagation Delay
- Pipe Diagrams
- Overloaded Links

Brief Preview of the Semester

IMPORTANT

Packet delay: four sources



The nodal processing delay (check bit errors, determine output link) is typically very small and can be ignored

$$d_{nodal} = d_{queue} + d_{trans} + d_{prop}$$

d_q: queueing delay

- time waiting at output link for transmission
- depends on congestion level of router

d_{tx}: transmission delay:

- L: packet length (bits)
- R: link *transmission rate (bps)*
- $d_{trans} = L/R$

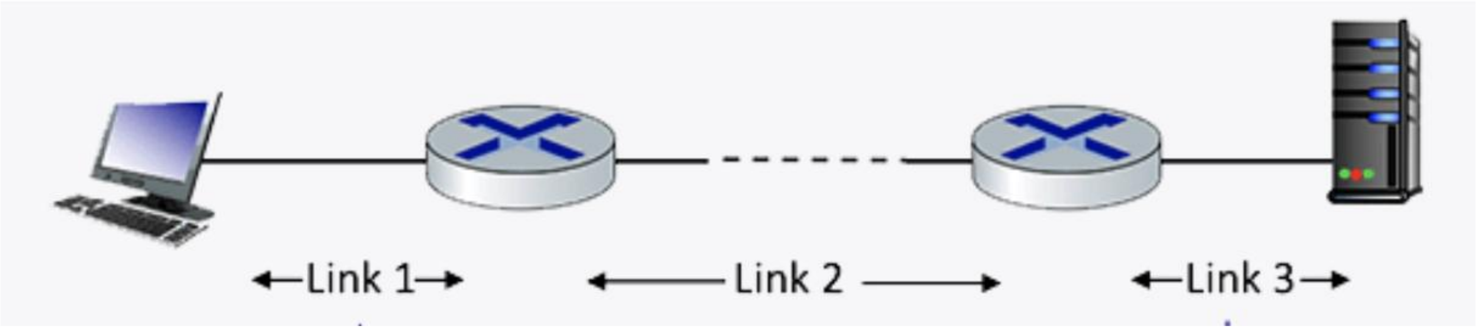
d_{prop}: propagation delay:

- d: length of physical link
- s: propagation speed (~2×10^8 m/sec)
- $d_{prop} = d/s$

Q1 End-to-end delay

Consider the network shown in the figure, with three links, each with a transmission rate of 1 Mbps, and a propagation delay of 1 msec per link. Assume the length of a packet is 1000 bits.

- What is the end-end delay of a packet from when it first begins transmission on link 1, until is it received in full by the server at the end of link 3. Assume that queueing delays are zero.



Q1 End-to-end delay ANS

- Packet length $L = 1000$bits, link rate $R = 1$Mbps $\longrightarrow$ transmission delay per link $t_{tx} = L/R = 1000/1{,}000{,}000 = 0.001$s = 1ms.
- Propagation delay per link t_{prop} = 1ms.
- Each link contributes $t_{tx} + t_{prop} = 1 + 1 =$ 2ms.
- Three links $\longrightarrow$ total $3 \times$ 2ms = 6ms.

Q2 End-to-end delay

Consider the scenario shown in Figure 1: a server is connected to a router by a 100Mbps link with a 50ms propagation delay. This router is connected to two other routers, each over a 50Mbps link with a 200ms propagation delay. A 1Gbps link connects each client to each of these routers with 0 propagation delay. (Ignore the cache.) All packets in the network are 20,000 bits long.

- What is the end-to-end delay (in ms) from when a packet is transmitted by the server to when it is received by the client? Assume there's no queuing delay at the routers.

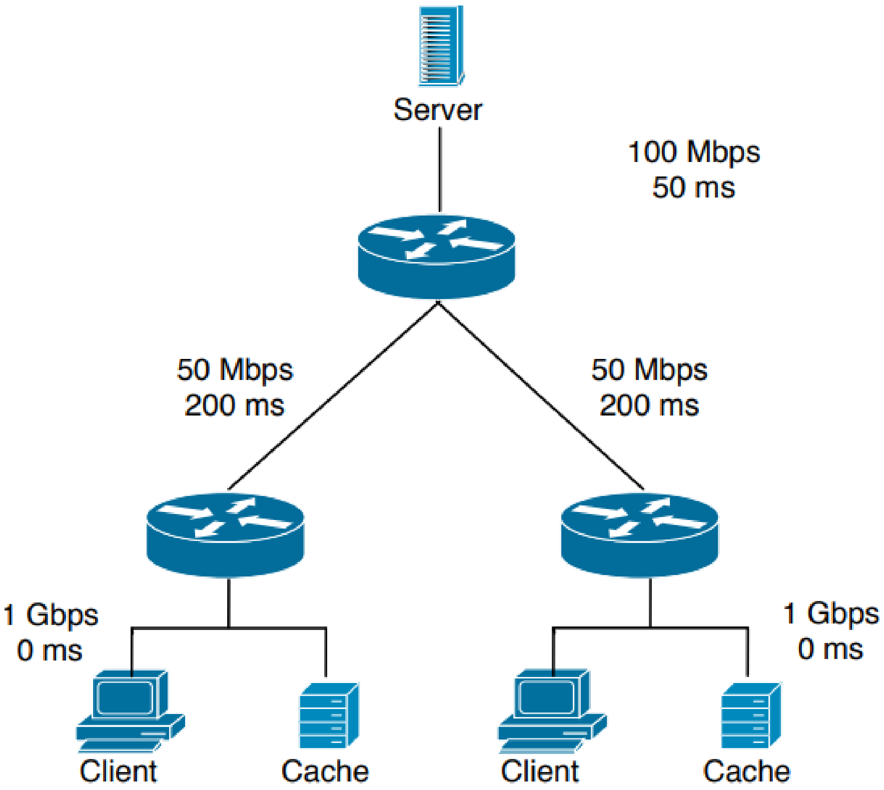


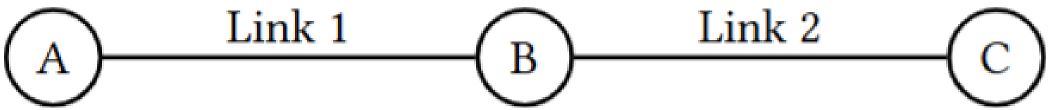
Figure 1

Q2 End-to-end delay ANS

- Transmission delays:
  - 100 Mbps link: $20{,}000/100{,}000{,}000 = 0.0002$s = 0.2ms
  - 50 Mbps link: $20{,}000/50{,}000{,}000 = 0.0004$s = 0.4ms
  - 1 Gbps link: $20{,}000/1{,}000{,}000{,}000 = 0.00002$s = 0.02ms
  - Total transmission delay: $0.2 + 0.4 + 0.02 = $ 0.62ms
- Propagation delays:
  - Server → router: 50 ms
  - Router → router: 200 ms
  - Router → client: 0 ms
  - Total propagation delay: $50 + 200 = 250$ms
- End-to-end delay: $250\ ms + 0.62\ ms = 250.62\ ms$

Q3.1 End-to-end delay

- In the diagram below, we have two different links, each with different physical properties.
  - Suppose T_1 = 10000, L_1 = 100000, and $S_1 = 2.5 \times 10^8$. How long does it take to send a 500-byte packet from Node A to Node B?



	Link 1	**Link 2**
Physical length of link	L_1 meters	L_2 meters
Speed of data propagation	S_1 meters/sec	S_2 meters/sec
Bandwidth of link	T_1 bits/sec	T_2 bits/sec

Q3.1 End-to-end delay ANS

The total time needed is the sum of the transmission delay to push the packet onto Link 1 and the propagation delay for the packet to travel from Node A to Node B.

$$t_{\text{total}} = t_{\text{transmission}} + t_{\text{propagation}}$$

$$t_{\text{total}} = \frac{\text{packet size}}{\text{transmission rate of Link 1}} + \frac{\text{distance between A and B}}{\text{propagation speed}}$$

$$t_{\text{total}} = \frac{500 \text{ bytes} \times 8 \frac{\text{bits}}{\text{byte}}}{10000 \frac{\text{bits}}{\text{second}}} + \frac{100000 \text{ meters}}{2.5 \times 10^8 \frac{\text{meters}}{\text{second}}}$$

$$t_{\text{total}} = 0.4s + 0.0004s = \boxed{0.4004\text{s}}$$

Notice that transmission delay dominates more than 99.9% in this case.

Q3.2 End-to-end delay

- The RTT (Round Trip Time) is the time it takes to send a packet (from source to destination) and receive a response (from destination to source). Count from the time the source transmits the first byte, to the time the source receives the last byte of the response.
- Node A sends a x-byte packet to Node C. Then, Node C sends an x-byte response back to Node A. What is the RTT for this exchange?
- Note: Since there is only one packet, there is no queuing delays. We assume processing delay is negligible, so Node C starts transmitting the response immediately after it receives the last byte of the packet.

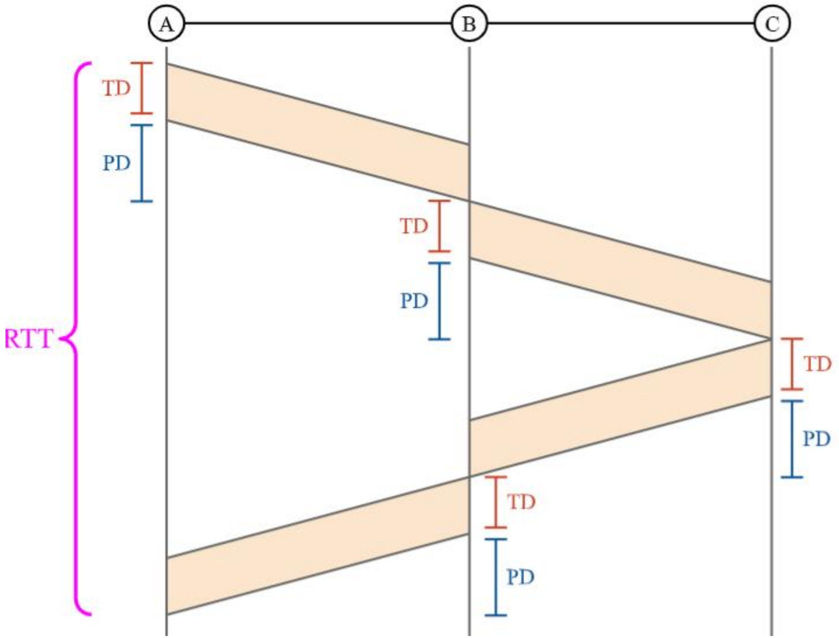
Q3.2 End-to-end delay ANS

Note the sequence of delays the packet experiences during its route from A to C

1. Transmission delay to push the packet onto Link 1.
2. Propagation delay as the packet travels from Node A to Node B.
3. Transmission delay to push the packet onto Link 2.
4. Propagation delay as the packet travels from Node B to Node C.
5. Transmission delay to push the packet onto Link 2.
6. Propagation delay as the packet travels from Node C to Node B.
7. Transmission delay to push the packet onto Link 1.
8. Propagation delay as the packet travels from Node B to Node A.

Summing these delays yields the total RTT:

$$\frac{8x}{T_1} + \frac{L_1}{S_1} + \frac{8x}{T_2} + \frac{L_2}{S_2} + \frac{8x}{T_2} + \frac{L_2}{S_2} + \frac{8x}{T_1} + \frac{L_1}{S_1}$$



Store-and-forward: A router can start forwarding a packet only after it has received all bytes of the packet.

Q3.3 End-to-end delay

- Node A sends two packets: Packet P_1 of size D_1 bytes. Packet P_2 of size D_2 bytes. Node A starts sending packet P_1 at $t = 0$. Node A immediately starts sending packet P_2 after it finishes transmitting all the bits of P_1.
- When will Node C receive the last bit of packet P_2?

There will be a queuing delay at Node B if packet P_2 arrives at Node B before packet P_1 is finished being pushed onto Link 2.

Let's start by computing the time at which P_1 finishes being pushed onto Link 2. P_1 takes $\frac{8D_1}{T_1}$ seconds to be pushed onto Link 1, $\frac{L_1}{S_1}$ seconds to propagate from Node A to Node B, and then $\frac{8D_1}{T_2}$ seconds to be pushed onto Link 2. Hence P_1 leaves Node B at time:

$$t_1 = \frac{8D_1}{T_1} + \frac{L_1}{S_1} + \frac{8D_1}{T_2}$$

Next, let's figure out the time when P_2 arrives at Node B. It first waits $\frac{8D_1}{T_1}$ seconds for P_1 to be completely pushed onto Link 1, then takes $\frac{8D_2}{T_1}$ seconds of transmission delay to be pushed onto Link 1 itself, before finally needing $\frac{L_1}{S_1}$ seconds of propagation delay to reach Node B. With this, we know that P_2 reaches Node B at time:

$$t_2 = \frac{8D_1}{T_1} + \frac{8D_2}{T_1} + \frac{L_1}{S_1}$$

There's queuing delay if $t_1 > t_2$, and the length of the delay can be expressed as:

$$t_1 - t_2 = \left(\frac{8D_1}{T_1} + \frac{L_1}{S_1} + \frac{8D_1}{T_2}\right) - \left(\frac{8D_1}{T_1} + \frac{8D_2}{T_1} + \frac{L_1}{S_1}\right) = \frac{8D_1}{T_2} - \frac{8D_2}{T_1}$$

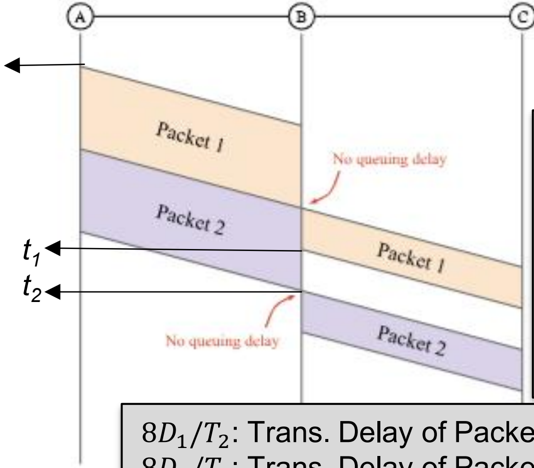
With this analysis in hand, we can express the time at which Node C receives the last bit of P_2 as follows:

$$t_{\text{total}} = \frac{8D_1}{T_1} + \frac{8D_2}{T_1} + \frac{L_1}{S_1} + \max\left(\left(\frac{8D_1}{T_2} - \frac{8D_2}{T_1}\right), 0\right) + \frac{8D_2}{T_2} + \frac{L_2}{S_2}$$

From left to right, the terms in this sum are:

1. The transmission delay to push P_1 onto Link 1.
2. The transmission delay to push P_2 onto Link 1.
3. The propagation delay as P_2 travels from Node A to Node B.
4. The queuing delay at Node B. Note that the use of the max operator allows us to express the two cases when there is and when there isn't queuing delay compactly.
5. The transmission delay to push P_2 onto Link 2.
6. The propagation delay as P_2 travels from Node B to Node C.

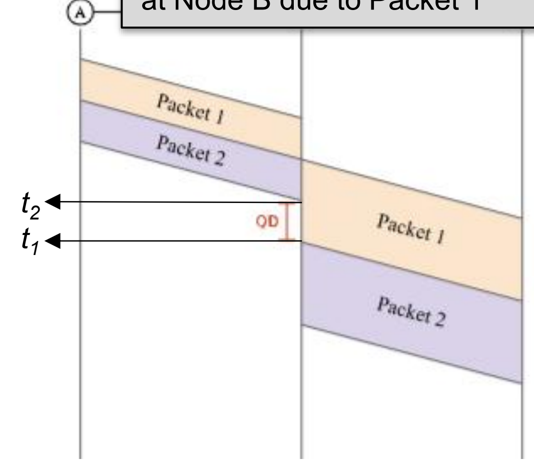
Below is the time-graph of a packet in flight without queuing delay:



t_1: when P_1 leaves Node B
t_2: when P_2 has fully arrived at Node B, and is ready to transmitted to Node C

$8D_1/T_2$: Trans. Delay of Packet 1 on Link 2
$8D_2/T_1$: Trans. Delay of Packet 2 on Link 1
$\max(\frac{8D_1}{T_2} - \frac{8D_2}{T_1}, 0)$: Queuing delay of Packet 2 at Node B due to Packet 1

And with queuing del



Q3.4 End-to-end delay

- Find the variable relations that need to be satisfied in order to have no queuing delays for part (c).

Q3.4 End-to-end delay ANS

From the analysis we conducted in the previous part, we know there will be queuing delays if $t_1 > t_2$, or $\frac{8D_1}{T_2} > \frac{8D_2}{T_1}$. Hence, there are no queuing delays if: $\frac{8D_1}{T_2} \leq \frac{8D_2}{T_1}$ After simplifying, we see the relation that must be satisfied is:

$$\boxed{\frac{D_1}{T_2} \leq \frac{D_2}{T_1}}$$

Q4. Statistical Multiplexing

- Consider three flows F_1, F_2, F_3 sending packets over a single link. The sending pattern of each flow is described by how many packets it sends within each one-second interval; the table below shows these numbers for the first ten intervals. A perfectly smooth (i.e., non-bursty) flow would send the same number of packets in each interval, but our three flows are very bursty, with highly varying numbers of packets in each interval:
- 1) What is the peak rate of each flow? What is the sum of the peak rates?
- 2) Now consider all packets to be in the same aggregate flow. What is the peak rate of this aggregate flow?
- 3) Which is higher - the sum of the peaks, or the peak of the aggregate?

Time (s)	1	2	3	4	5	6	7	8	9	10
F_1	1	8	3	15	2	1	1	34	3	4
F_2	6	2	5	5	7	40	21	3	34	5
F_3	45	34	15	5	7	9	21	5	3	34

Q4. Statistical Multiplexing ANS

- 1) The peak rate is the highest the flow gets throughout the whole period. The peak rate of F_1 is 34, of F_2 is 40, and of F_3 is 45.The sum of their peaks is 34 + 40 + 45 = 119.
- 2) Summing the flows together column by column, we get the following values for an aggregate flow. The peak of the aggregate flow happens at 1s, where it is 52.
- 3) The sum of the peaks is 119, whereas the peak of the aggregate is 52, so the sum of the peaks is much higher. This is the insight from Statistical Multiplexing. The peak of the aggregate can only be at most the sum of the peaks, but that only happens in the case that all of the peaks happen at the same time. This is very unlikely, so usually, the peak of the aggregate is much lower than the sum of the peaks.

Time (s)	1	2	3	4	5	6	7	8	9	10
F_1	1	8	3	15	2	1	1	34	3	4
F_2	6	2	5	5	7	40	21	3	34	5
F_3	45	34	15	5	7	9	21	5	3	34