

CSC 163: Computing, Ethics, and Society

Chapter 10

Utility Functions Exercises Solution

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Exercise: Expected value vs utility

Utility over wealth is $u(x) = \ln x$. Compare a lottery (50 % \$1, 50 % \$100) with sure amounts.

(a) Fill in the table: expected value (EV) and expected utility (EU).

(b) Does this agent prefer the lottery or a sure \$20? Which risk attitude is that?

(c) What sure amount (certainty equivalent) is worth the same as the lottery? How much EV is given up?

Option	EV	EU
Lottery: 50 % \$1 / 50 % \$100		
Sure \$20	20	
Sure \$10	10	

Exercise Solution: Expected value vs utility

- (a) Lottery: $EV = 0.5 + 50 = 50.5$; $EU = 0.5 \ln 1 + 0.5 \ln 100 = 2.30$. Sure \$20: $EV = 20$, $EU = \ln 20 = 3.00$. Sure \$10: $EU = \ln 10 = 2.30$.
- (b) A sure \$20 ($EU = 3.00 > 2.30$), despite the lower EV: **risk averse** (concave utility).
- (c) In CE = 2.30, so CE = **\$10**; the agent gives up $50.5 - 10 = 40.5$ of EV to avoid risk.

Option	EV	EU
Lottery: 50 % \$1 / 50 % \$100	50.5	2.30
Sure \$20	20	3.00
Sure \$10	10	2.30

Exercise: Risk attitudes

A lottery pays \$0 or \$100 with probability 0.5 each; the alternative is a sure \$50. For each utility function, compute $EU(\text{lottery})$ and $u(\$50)$, state the preferred option and the agent's risk attitude (averse, neutral, seeking).

$u(x)$	$EU(\text{lottery})$	$u(\$50)$	Preferred	Attitude
$\sqrt{x}$				
x				
x^2				

Exercise Solution: Risk attitudes

Concave $u \rightarrow$ risk averse; linear $\rightarrow$ risk neutral; convex $\rightarrow$ risk seeking. The EV of the lottery is \$50 in all cases: only the shape of u differs.

$u(x)$	EU(lottery)	$u(\$50)$	Preferred	Attitude
$\sqrt{x}$	5	7.07	Sure \$50	Risk averse
x	50	50	Indifferent	Risk neutral
x^2	5,000	2,500	Lottery	Risk seeking

Exercise: Allais paradox

Scenario A: A1 = \$1M for sure; A2 = 89 % \$1M, 10 % \$5M, 1 % \$0. Scenario B: B1 = 11 % \$1M, 89 % \$0; B2 = 10 % \$5M, 90 % \$0.

Many people choose A1 and B2.

(a) Fill in the expected values. Which gambles does EV favor?

(b) Normalize $u(\$0) = 0$, $u(\$1M) = 1$, $u(\$5M) = x$. For what x is A1 preferred to A2? For what x is B2 preferred to B1?

(c) Can an expected-utility maximizer choose both A1 and B2? Which axiom is violated?

Gamble	EV (\$)
A1	
A2	
B1	
B2	

Exercise Solution: Allais paradox

- (a) EV: A1 = \$1,000,000; A2 = \$1,390,000; B1 = \$110,000; B2 = \$500,000. EV favors **A2 and B2**.
- (b) A1 > A2: $1 > 0.89 + 0.10x \rightarrow x < 1.1$. B2 > B1: $0.10x > 0.11 \rightarrow x > 1.1$.
- (c) **No**: no x satisfies both. A1 and B2 together violate the **Independence axiom** (removing the common 89 % outcome makes the pairs identical).

Gamble	EV (\$)
A1	1,000,000
A2	1,390,000
B1	110,000
B2	500,000

Exercise: True or false

For each statement write T or F. If it is false, correct it with a few words.

Statement	T / F	Correction (if false)
A corrigible AI accepts being shut down or having its utility function changed		
A coffee-fetching AI loses utility when corrected, so it has a reason to ignore the correction		
With $\log_2$ utility the St. Petersburg game has infinite expected utility		
A risk-averse agent has a convex utility function		
Prospect theory: risk-seeking in gains and risk-averse in losses		
The independence axiom of vNM is uncontroversial		
In the 600-people example, "save 200 for sure" and "400 die for sure" are different outcomes		

Exercise Solution: True or false

Each statement: 1 point for T/F, 1 point for the correction.

Statement	T / F	Correction (if false)
A corrigible AI accepts being shut down or having its utility function changed	T	—
A coffee-fetching AI loses utility when corrected, so it has a reason to ignore the correction	T	—
With $\log_2$ utility the St. Petersburg game has infinite expected utility	F	EV is infinite, but the expected utility is 2
A risk-averse agent has a convex utility function	F	A concave utility function (convex = risk seeking)
Prospect theory: risk-seeking in gains and risk-averse in losses	F	Reverse: risk-averse in gains, risk-seeking in losses; steeper for losses
The independence axiom of vNM is uncontroversial	F	Independence and decomposability are contentious
In the 600-people example, "save 200 for sure" and "400 die for sure" are different outcomes	F	They are identical: only the framing (gains vs losses) differs