

CSC 144V: Autonomous Vehicles

Lecture 7

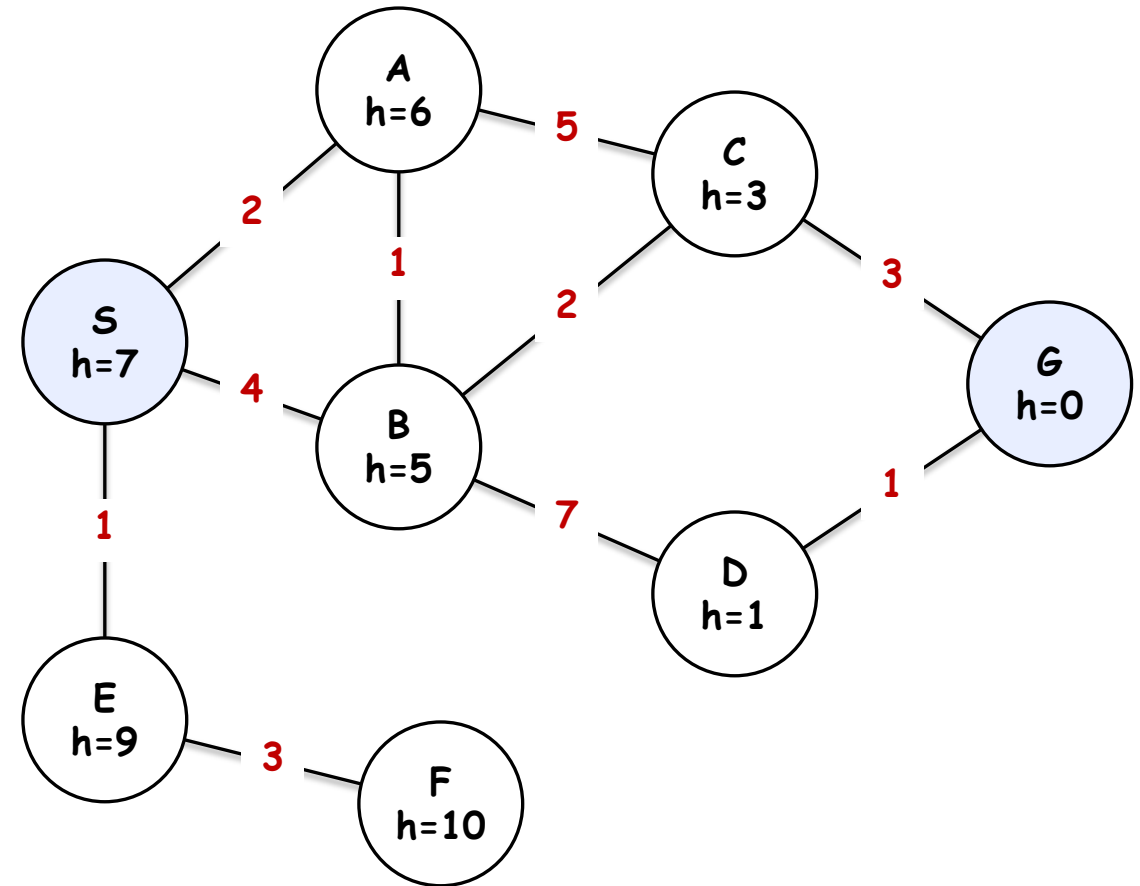
Planning Exercises Solution

Department of Computer Science,
Hofstra University

Exercise: Dijkstra vs. A*

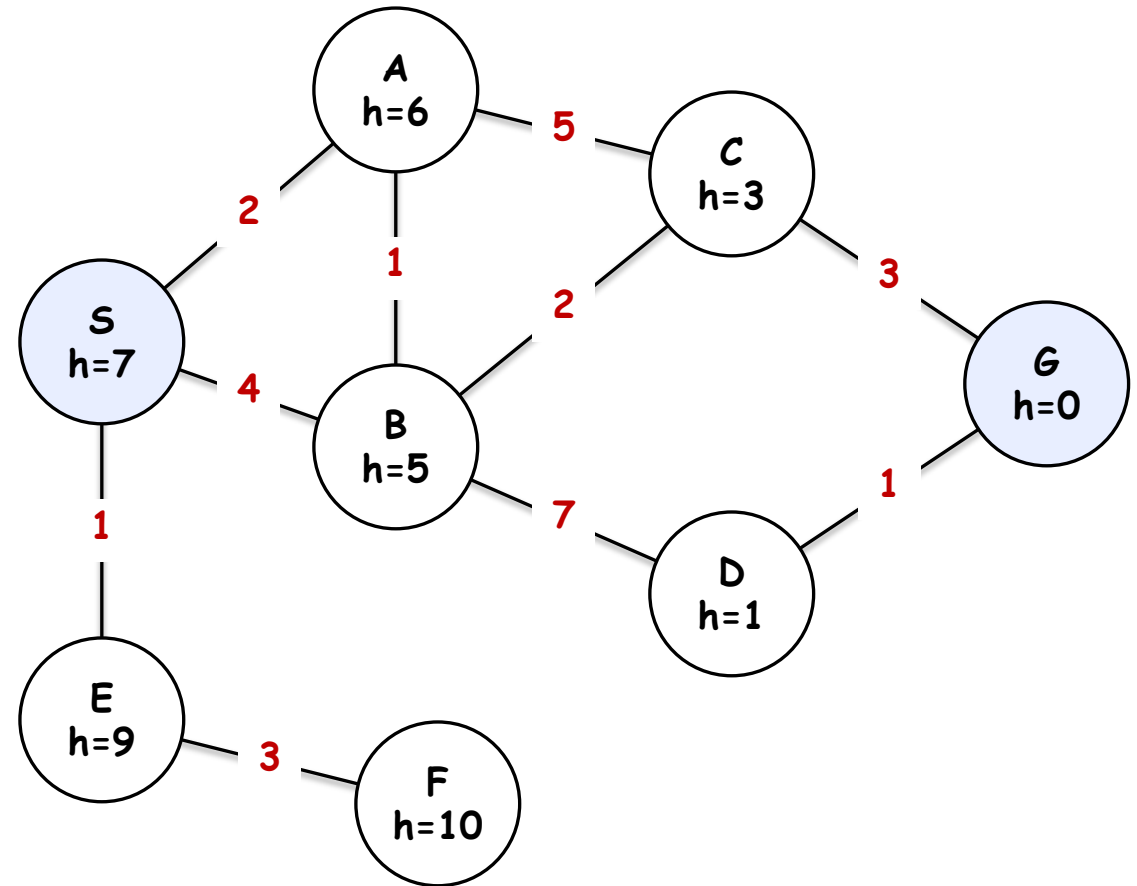
Find the shortest path $S \rightarrow G$. Edge costs are on the edges; the heuristic h (straight-line estimate to G) is in each node.

- (a) Dijkstra: list the nodes in expansion order with their g values; give the path and its cost.
- (b) A*: list the nodes in expansion order with $f = g + h$.
- (c) Is h admissible? How many nodes does each algorithm expand?



Exercise Solution: Dijkstra vs. A*

- **Dijkstra** (g): S(0), E(1), A(2), B(3), F(4), C(5), G(8) → **7** nodes; path $S \rightarrow A \rightarrow B \rightarrow C \rightarrow G$, cost **8**.
- **A*** ($f = g + h$): S(7), A(8), B(8), C(8), G(8) → **5** nodes; same path. E ($f = 10$) and F are never expanded.
- h is **admissible**: it never overestimates the true cost-to-go (e.g. E: $9 \leq 9$, F: $10 \leq 12$, A: $6 \leq 6$).
- A* is directed towards the goal and expands fewer nodes.



Exercise: Behavior FSM trace

Track Speed → Follow Leader if gap < 15 m; → Decelerate to Stop if $d \leq 30$ m (d = distance to stop line).

Follow Leader → Track Speed if gap ≥ 15 m. **Decelerate to Stop** → Stop if $v \leq 0.2$ m/s and $d \leq 1$ m.

Stop → Track Speed if stopped ≥ 3 s and the intersection is clear. Start in Track Speed.

(a) Trace the state after each step. (b) Why $v \leq 0.2$ instead of $v == 0$? (c) Why is there no transition Decelerate to Stop → Track Speed at a stop sign?

Step	Inputs	State after step
1	$d = 80$ m, $v = 12$ m/s, no leader	
2	$d = 60$ m, $v = 12$ m/s, leader gap 12 m	
3	$d = 45$ m, $v = 10$ m/s, no leader	
4	$d = 28$ m, $v = 9$ m/s, no leader	
5	$d = 3$ m, $v = 1.0$ m/s	
6	$d = 0.5$ m, $v = 0.1$ m/s	
7	stopped 3 s, intersection not clear	
8	stopped 4 s, intersection clear	

Exercise Solution: Behavior FSM trace

- (b) Speed estimates are noisy: $v == 0$ may never hold; a threshold is more robust.
- (c) At a stop sign the vehicle must come to a complete stop, so the only exit is via Stop (at a traffic light such a transition would exist).

Step	Inputs	State after step
1	$d = 80$ m, $v = 12$ m/s, no leader	Track Speed
2	$d = 60$ m, $v = 12$ m/s, leader gap 12 m	Follow Leader
3	$d = 45$ m, $v = 10$ m/s, no leader	Track Speed
4	$d = 28$ m, $v = 9$ m/s, no leader	Decelerate to Stop
5	$d = 3$ m, $v = 1.0$ m/s	Decelerate to Stop
6	$d = 0.5$ m, $v = 0.1$ m/s	Stop
7	stopped 3 s, intersection not clear	Stop
8	stopped 4 s, intersection clear	Track Speed

Exercise: Speed profile

Speed profile generation for a planned path.

- Linear ramp: accelerate from 0 to $v_{\text{ref}} = 20 \text{ m/s}$ with $a = 2 \text{ m/s}^2$.
- A curve of radius $R = 50 \text{ m}$ must be taken with lateral acceleration $v^2/R \leq 3 \text{ m/s}^2$.
- Before the curve the car decelerates from 20 m/s at 4 m/s^2 .

(a) Time and distance of the ramp.

(b) Maximum speed in the curve.

(c) How far before the curve must braking start, and how long does it take?

(d) Trapezoidal stop at a red light from 20 m/s at 4 m/s^2 : distance and time?

Quantity	Answer
Ramp time / distance	
Max speed in curve	
Braking before curve	
Stop from 20 m/s	

Exercise Solution: Speed profile

- (a) $t = 20/2 = 10 \text{ s}$; $d = \frac{1}{2} \cdot 2 \cdot 10^2 = 100 \text{ m}$.
- (b) $v_{\text{max}} = \sqrt{3 \times 50} = \sqrt{150} \approx 12.2 \text{ m/s}$ ($\approx 44 \text{ km/h}$).
- (c) $d = (20^2 - 12.25^2)/(2 \times 4) = 250/8 \approx 31 \text{ m}$ before the curve; $t = (20 - 12.25)/4 \approx 1.9 \text{ s}$.
- (d) $d = 20^2/(2 \times 4) = 50 \text{ m}$; $t = 20/4 = 5 \text{ s}$.
- A speed profile must respect curvature (lateral acceleration) as well as longitudinal limits.

Quantity	Answer
Ramp time / distance	10 s / 100 m
Max speed in curve	12.2 m/s
Braking before curve	31 m / 1.9 s
Stop from 20 m/s	50 m / 5 s

Exercise: RSS safe distance

RSS minimum safe longitudinal distance (rear car = ego, front car brakes hard):

$$d_{\min} = [v_r \rho + \frac{1}{2} a_{\text{acc}} \rho^2 + (v_r + \rho a_{\text{acc}})^2 / (2 b_{\min}) - v_f^2 / (2 b_{\max})]_+$$

- $v_r = v_f = 20 \text{ m/s}$; response time $\rho = 0.5 \text{ s}$
- $a_{\text{acc}} = 3 \text{ m/s}^2$ (rear may accelerate during ρ); $b_{\min} = 6 \text{ m/s}^2$ (rear brakes); $b_{\max} = 8 \text{ m/s}^2$ (front brakes)

(a) Compute $d_{\min}$. (b) The actual gap is 20 m: safe? What is the proper response?

(c) Recompute $d_{\min}$ for $\rho = 1 \text{ s}$. Is a 30 m gap safe for $\rho = 0.5 \text{ s}$ and for $\rho = 1 \text{ s}$?

Exercise Solution: RSS safe distance

- (a) $20 \times 0.5 = 10$; $\frac{1}{2} \cdot 3 \cdot 0.25 = 0.375$; $(20 + 1.5)^2/12 = 38.52$; $20^2/16 = 25 \rightarrow d_{\min} = 10 + 0.375 + 38.52 - 25 \approx \mathbf{23.9 \text{ m}}$.
- (b) $20 \text{ m} < 23.9 \text{ m}$: **non-safe**. Proper response: **brake** (at least $b_{\min} = 6 \text{ m/s}^2$) until the distance is safe again; the blame time is the first moment the situation became dangerous.
- (c) $\rho = 1 \text{ s}$: $20 + 1.5 + 23^2/12 - 25 = 20 + 1.5 + 44.08 - 25 \approx \mathbf{40.6 \text{ m}}$. A 30 m gap is safe for $\rho = 0.5 \text{ s}$ ($30 > 23.9$) but **not** for $\rho = 1 \text{ s}$ ($30 < 40.6$).
- Safety vs. efficiency: slower reaction needs larger gaps.