

CSC 144V: Autonomous Vehicles

Lecture 6

LiDAR Perception Exercises Solution

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Exercise: Time of flight and spacing

$c = 3 \times 10^8$ m/s; range $R = c \cdot \Delta t / 2$; max pulse rate per channel $f \leq c / (2R_{\text{max}})$; neighboring point spacing $s = R \cdot \tan(\Delta\theta)$.

Fill in the table: $\Delta\theta = 0.2^\circ$, $R_{\text{max}} = 200$ m.

Quantity	Answer
Return time of a 200 m target	
Timing accuracy for 5 cm range accuracy	
Max pulse rate per channel	
Point spacing at 50 m / 100 m	
Range where spacing = 0.5 m	

Exercise Solution: Time of flight and spacing

- $\Delta t = 2R/c = 400/(3 \times 10^8) = \mathbf{1.33 \mu s}$.
- $5 \text{ cm} \rightarrow \Delta t = 2 \times 0.05/c = \mathbf{0.33 \text{ ns}}$ ($1 \text{ ns} \approx 15 \text{ cm}$).
- $f \leq c/(2R_{\text{max}}) = 3 \times 10^8/400 = \mathbf{750 \text{ kHz}}$.
- $s(50 \text{ m}) = 50 \tan 0.2^\circ = \mathbf{0.17 \text{ m}}$; $s(100 \text{ m}) = \mathbf{0.35 \text{ m}}$.
- $R = 0.5/\tan 0.2^\circ \approx \mathbf{143 \text{ m}}$: beyond it a 0.5 m pedestrian gets < 1 beam per scan line.

Quantity	Answer
Return time of a 200 m target	$\mathbf{1.33 \mu s}$
Timing accuracy for 5 cm range accuracy	$\mathbf{0.33 \text{ ns}}$
Max pulse rate per channel	$\mathbf{750 \text{ kHz}}$
Point spacing at 50 m / 100 m	$\mathbf{0.17 \text{ m} / 0.35 \text{ m}}$
Range where spacing = 0.5 m	$\mathbf{\approx 143 \text{ m}}$

Exercise: Coordinates and distortion

A return has range $r = 20$ m, azimuth $\theta = 30^\circ$, elevation $\phi = -5^\circ$. The LiDAR is mounted 1.8 m above the road. $x = r \cos\phi \cos\theta$, $y = r \cos\phi \sin\theta$, $z = r \sin\phi$.

- (a) Compute (x, y, z) . Is this likely a ground point?
- (b) The vehicle drives at 20 m/s and the LiDAR sweeps at 10 Hz. How far does the vehicle move during one sweep? Why does it matter?
- (c) An extrinsic rotation error of 0.5° shifts a point at 100 m by how much?

Exercise Solution: Coordinates and distortion

- (a) $x = 20 \cdot 0.996 \cdot 0.866 \approx \mathbf{17.25\ m}$, $y = 20 \cdot 0.996 \cdot 0.5 \approx \mathbf{9.96\ m}$, $z = 20 \cdot \sin(-5^\circ) \approx \mathbf{-1.74\ m}$. $z \approx -1.8\ m$ (the road height below the sensor) $\rightarrow$ **likely a ground point**.
- (b) $20 \times 0.1 = \mathbf{2\ m}$. First and last points of a sweep see different poses: **deskew** each point with the pose at its timestamp, otherwise walls blur and a seam appears.
- (c) $100 \cdot \tan 0.5^\circ \approx \mathbf{0.87\ m}$: calibration errors grow with range.

Exercise: RANSAC iterations

RANSAC samples a minimal set of s points, counts inliers, and repeats N times. For success probability $p = 0.99$ and inlier ratio w :

$$N = \ln(1 - p) / \ln(1 - w^s)$$

(a) Fill in the table (round up).

(b) Why does the ground-plane fit sample only points with $z < 0.5$ m?

Model	s	w	N
Plane	3	0.5	
Plane	3	0.3	
Plane	3	0.8	
Line	2	0.5	

Exercise Solution: RANSAC iterations

- (a) $w = 0.5, s = 3: \ln 0.01 / \ln 0.875 = 34.5 \rightarrow 35$. $w = 0.3: 168.2 \rightarrow 169$. $w = 0.8: 6.4 \rightarrow 7$. A line ($s = 2$) with $w = 0.5: 16.01 \rightarrow 17$.
- (b) It raises the inlier ratio w (fewer walls and cars among the samples) and makes the fitted plane the **road**, not a wall; a lower w makes N explode.

Model	s	w	N
Plane	3	0.5	35
Plane	3	0.3	169
Plane	3	0.8	7
Line	2	0.5	17

Exercise: DBSCAN by hand

DBSCAN with $\epsilon = 1.0$ m and **minPts = 3** (the point itself counts). Count the neighbors within ϵ of each point, classify it as core, border or noise, and give its cluster.

Point	(x, y)	# within ϵ	Type	Cluster
P1	(0, 0)			
P2	(0.8, 0)			
P3	(1.6, 0.2)			
P4	(2.4, 0.9)			
P5	(4, 2)			
P6	(6, 1)			
P7	(6.7, 1.1)			
P8	(6.3, 1.8)			
P9	(7.5, 1.3)			

Exercise Solution: DBSCAN by hand

Core: ≥ 3 points within ϵ (incl. itself). Border: not core, but within ϵ of a core point. Noise: neither. P4 is 1.06 m from P3 $\rightarrow$ just outside $\epsilon \rightarrow$ noise.

Point	(x, y)	# within ϵ	Type	Cluster
P1	(0, 0)	2	border	1
P2	(0.8, 0)	3	core	1
P3	(1.6, 0.2)	2	border	1
P4	(2.4, 0.9)	1	noise	-
P5	(4, 2)	1	noise	-
P6	(6, 1)	3	core	2
P7	(6.7, 1.1)	4	core	2
P8	(6.3, 1.8)	3	core	2
P9	(7.5, 1.3)	2	border	2

Exercise: PointNet and pillars

(a) PointNet. Three points have per-point features $h(x_1) = (1, 5)$, $h(x_2) = (3, 2)$, $h(x_3) = (2, 4)$. The global feature is the max over points. Compute it for the order (x_1, x_2, x_3) and for (x_3, x_1, x_2) . Why a symmetric function?

(b) Sparsity. Region $80 \text{ m} \times 80 \text{ m}$; one sweep has $\approx 190,000$ points. Voxels 0.1 m (z: 4 m height) vs. pillars 0.16 m (no z split). Compute the number of cells and the maximum occupied fraction (pillars: $P = 12,000$ non-empty).

Representation	Cells	Occupied
Voxels 0.1 m		
Pillars 0.16 m		

Exercise Solution: PointNet and pillars

- (a) $\max = (\max(1, 3, 2), \max(5, 2, 4)) = (3, 5)$ in both orders: the max is symmetric, so the output does not depend on the order of the $N!$ possible orderings.
- (b) Voxels: $800 \times 800 \times 40 = 25.6 \text{ M}$ cells; $\leq 0.19 \text{ M} / 25.6 \text{ M} \approx 0.74 \%$ occupied $\rightarrow$ dense 3D conv wastes $> 99 \%$ of its work on empty space.
- Pillars: $500 \times 500 = 250,000$; $12,000 / 250,000 = 4.8 \%$ occupied. Pillar tensor $(P, N, D) = (12000, 100, 9)$ float32 = 10.8 M values $\approx 43 \text{ MB}$; 2D convolutions only.

Representation	Cells	Occupied
Voxels 0.1 m	25.6 M	$\leq 0.74 \%$
Pillars 0.16 m	250,000	$\leq 4.8 \%$